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Some digraph classes that meet the directed path partition conjecture. (English)

Zbl 1569.05117

Discrete Appl. Math. 372, 260-267 (2025).

Let D be a digraph, and $\lambda(D)$ be the length of a longest directed path in D . The directed path partition conjecture (DPPC) says that for every $q \in \{1, 2, \dots, \lambda(D) - 1\}$ there is a partition (A, B) of $V(D)$ such that $\lambda(D[A]) \leq q$ and $\lambda(D[B]) \leq \lambda(D) - q$. This paper proves the DPPC for certain graph classes.

A digraph D is said to be k -quasi-transitive if, whenever there is a directed path of length k from u to v , then we have that u and v are adjacent in D . Given a digraph H with vertex set $\{u_1, \dots, u_t\}$, and digraphs $D_1, \dots, D_t$, the composition over H with $D_1, \dots, D_t$ is the digraph $H[D_1, \dots, D_t]$ defined as the disjoint union of $D_1, \dots, D_t$ along with the arcs (v, w) where $v \in V(D_i)$ and $w \in V(D_j)$ such that (u_i, u_j) is an arc of H .

In the first half of the paper, the authors show that non-strong 3-quasi-transitive digraphs and strong 4-transitive digraphs satisfy the DPPC. The authors use the structural characterizations of such graphs by *H. Galeana-Sánchez* et al. [Discrete Math. 310, No. 19, 2495–2498 (2010; Zbl 1213.05112)], *S. Wang* and *R. Wang* [ibid. 311, No. 4, 282–288 (2011; Zbl 1222.05090)] and *C. Hernández-Cruz* [Discuss. Math., Graph Theory 33, No. 2, 247–260 (2013; Zbl 1293.05136)] to prove their results. From the result of *A. Arroyo* and *H. Galeana-Sánchez* [Discrete Math. 313, No. 3, 293–300 (2013; Zbl 1256.05085)] that strong 3-quasi-transitive digraphs satisfy the DPPC, the conjecture is now known to be satisfied by all 3-quasi-transitive digraphs.

In the second half of the paper, the authors prove two results on compositions over unicyclic digraphs. First, let T be a unicyclic digraph on the vertex set $\{v_1, \dots, v_n\}$ with $C = v_1 v_2 \dots v_t v_1$ being the unique cycle in T , and let $D_1, \dots, D_n$ be digraphs such that $D_1, \dots, D_t$ have the same order, and $D_{t+1}, \dots, D_n$ each satisfy the DPPC. Then, $T[D_1, \dots, D_n]$ satisfies the DPPC. Second, if $C = v_1 v_2 \dots v_t v_1$ is a cycle and $D_1, \dots, D_t$ are any digraphs, then $C[D_1, \dots, D_t]$ satisfies the DPPC. These results generalize an earlier result of Arroyo-Galeana-Sánchez [loc. cit.] on sufficient conditions for a composition to satisfy the DPPC.

Reviewer: [Brahadeesh Sankarnarayanan \(Jodhpur\)](#)

MSC:

- [05C20](#) Directed graphs (digraphs), tournaments
- [05C38](#) Paths and cycles
- [05C70](#) Edge subsets with special properties (factorization, matching, partitioning, covering and packing, etc.)
- [05C12](#) Distance in graphs

Keywords:

[longest path](#); [3-quasi-transitive graph](#); [unicyclic graph](#); [composition](#); [digraph](#); [path partition conjecture](#)

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